Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind,
Pune – 16
Odd Semester Examination: Nov/Dec 2023-2024
Faculty: Science and Technology

Program: BSc Comp05 **Semester: III** **Set : B**
Program (Specific):B.Sc. Computer Science **Course Type: Core**
Class: S.Y.B.Sc(Comp. Sci.) **Max. Marks: 35**
Name of the Course: Groups and Coding Theory **Course Code: 23-MTC-231**
Paper no.: I **Time: 2Hrs**

Instructions to the candidate:

1) ***There are 3 sections in the question paper. Write each section on separate page.***
2) ***All Sections are compulsory.***
3) ***Figures to the right indicate full marks.***
4) ***Draw a well labelled diagram wherever necessary.***

SECTION: A

Q1) Solve any 5 of the following. **(10 Marks)**

a) Find the congruence class of $\bar{3}$ in $(\mathbb{Z}_8, +_8)$.

b) On $(\mathbb{R}, *)$, consider a*b = a + b + 2 . Find the identity element with respect to *.

c) Define $e: B^2 \rightarrow B^6$ be a (2,6) encoding function as follows
e(00) = 000000 , e(01) = 111000, e(10) = 100111, e(11) = 110011.
Find the minimum distance of e.

d) Define: Subgroup of a group.

e) Find suitable integers m and n such that 31 m + 13 n = 1.

f) Find the hamming distance between x = 0011110 and y=1010101. Also, find weights of x and y.

g) Prepare the composition table of $\mathbb{Z}_4$ with respect to addition modulo 4.

SECTION: B

Q.2) Solve any 3 of the following. (Marks 15)

a) If a, b, c, d $\in \mathbb{Z}$; n$\in \mathbb{N}$ and a $\equiv$ b (mod n), c $\equiv$ d (mod n) then prove that

i) (a + c) $\equiv$ (b + d) (mod n)

ii) ac$\equiv$ bd (mod n)

b) Let $G = (\mathbb{Z}_{10}, +_{10})$ and H= $\{ \bar{0}, \bar{5} \}$. Find all Cosets of H in $\mathbb{Z}_{10}$.

c) Prove that if a group < G , * > is with the property that square of every element is identity, then G is abelian.

d) Decode the following received words, for the following parity check matrix H,

where, $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

i) 111101 ii) 100100

e) Let σ =(1,5,2)(4,3) , τ = (6,1,9,7) be two permutations in S_7.Compute $\sigma\,\tau$ and $\sigma\sigma^{-1}$.

Also determine whether σ is even or odd?

SECTION: C

Q.3) Solve any 1 of the following. (Marks 10)

a) Let p = 13, q = 7, s = 25. Apply RSA method to encode the message "HI".

b) i) Show that G= $\{ 3^n / n\in \mathbb{Z} \}$ forms a group under multiplication. Is G abelian?

ii) Find GCD of 192 and 270. Also find x and y such that gcd (192, 270) = 192 x + 270 y.

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